Controlling one-way quantum steering in a modulated optomechanical system

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We investigate in detail the properties of the stationary quantum steering of two mechanical modes, where the two mechanical modes interact with two coupling cavity modes, and two four-tone driving lasers are used to pump the two cavity modes. By controlling the pumping lasers, the two cavity modes can act as two engineered reservoirs to cool the two mechanical modes to a squeezed state. When the damping rates of the two mechanical modes are different, numerical simulation results show that there are parameter regions where the state of the two mechanical modes only has one-way quantum steering, which is the most prominent feature of quantum steering from quantum entanglement.

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I. INTRODUCTION

The well-known Einstein-Podolsky-Rosen (EPR) paradox was proposed in a seminal paper to question the completeness of quantum mechanics [1]. Subsequently, Schrödinger introduced the term "entanglement" to discuss the EPR paradox, which implies the existence of steering [2,3].

The steering is intrinsically distinct from quantum entanglement and Bell nonlocality for it has asymmetric characteristics between the parties involved. Recently, Wiseman *et al.* proved with Werner states and isotropic states that the steering is a nonclassical correlation, stronger than entanglement but weaker than Bell nonlocality [4]. A lot of work is devoted to improving our understanding on quantum steering by giving qualitative judge criteria to decide whether a quantum state is steerable [5–9] or quantifying the steerability of a given quantum state [10,11]. Apart from its fundamental physical significance, the quantum steering has important practical applications, such as the quantum subchannel discrimination problem [12], and high-fidelity heralded teleportation using minimally entangled yet steerable resources [13]. Remarkably, the genuine one-way steering, which is the most prominent feature of steering, has been experimentally observed [14–20].

The steerable states of macroscopic and massive objects can be used for testing the fundamental principles of quantum mechanics and implementing quantum information processing, and much work has been devoted to finding effective ways to achieve these goals in cavity optomechanical systems [21–28]. The related works include the generation of quantum steering between a mechanical oscillator and an optical pulse [21,23] or between two massive mirrors [25], and the generation of hybrid atom-mechanical quantum steering in the steady-state regime [22,26]. In addition, one-way quantum steering between two electromagnetic fields mediated by a mechanical oscillator [24] or between the intracavity field and the mechanical oscillator [27] is also investigated.

It is noted that much progress has been made in the preparation of macroscopic entangled states in cavity optomechanical systems. Moreover, it is worth emphasizing that an experimental demonstration of entanglement between two macroscopic-scale mechanical oscillators has been realized [29] which is based on a series of proposals for using reservoir engineering [30–35]. The entanglement obtained by reservoir engineering can be significantly larger than that with other methods, and the method of reservoir engineering does not depend on the initial states. These advantages motivate us to consider reservoir-engineered quantum steering generation in cavity optomechanical systems.

Here, we propose a scheme for achieving steady-state one-way quantum steering between two mechanical oscillators. In the scheme two four-tone driving lasers are used to excite two coupling cavity modes which interact with two mechanical modes, respectively. By selecting appropriate driving lasers and mechanical damping rates, one-way quantum steering between two mechanical modes can be observed. Our method is a reservoir-engineered one-way quantum steering generation method. It does not depend on the initial states of the system. Such an advantage can lower the experimental difficulty, as demonstrated in entanglement generation. Compared with the steering generation proposal in Ref. [28], our scheme does not need additional squeezed light to drive the two cavities.

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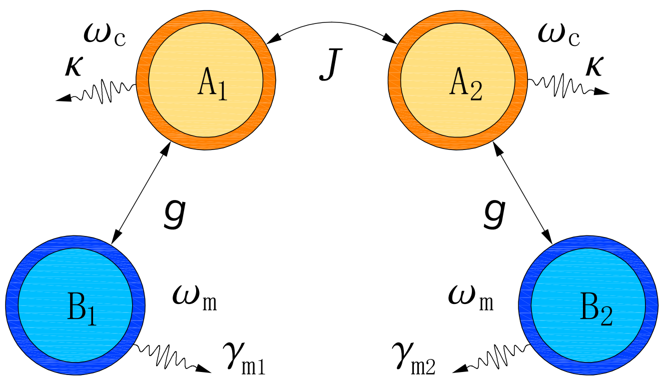

FIG. 1. Schematic representation of the system considered. Two phonon modes B_1 and B_2 with the same frequency ω_m respectively interact with two photon modes A_1 and A_2, which in turn are coupled via the photon tunneling.

This paper is organized as follows. In Sec. II, we introduce the fundamental model of our proposal. Then we study the steady-state entanglement and quantum steering between two mechanical oscillators in Sec. III. Conclusions are given in Sec. IV.

II. THE MODEL AND EFFECTIVE HAMILTONIAN

The sketch of the system is depicted in Fig. 1. Two phonon modes, B_1 and B_2, with the same frequency ω_m, respectively interact with two photon modes A_1 and A_2, which in turn are coupled via the photon tunneling. The two mechanical oscillators are assumed to be symmetrical except with different damping rates, i.e., they have different mechanical quality factors ω_m/γ_{m1} and ω_m/γ_{m2}. Both cavity modes with frequency ω_c are driven by external lasers with frequency ω_L and time-modulated amplitude $E(t)$. In a rotating frame with respect to laser frequency ω_L, the system Hamiltonian can be written as ($\hbar = 1$)

$$H = \sum_{j=1,2}[\Delta_0 A_j^\dagger A_j + \omega_m B_j^\dagger B_j - gA_j^\dagger A_j(B_j + B_j^\dagger) + iE(t)A_j^\dagger - iE(t)^* A_j] + J(A_1A_2^\dagger + A_1^\dagger A_2), \tag{1}$$

where A_j ($A_j^\dagger$) and B_j ($B_j^\dagger$) denote the annihilation (creation) operators of the jth cavity mode and phonon mode, respectively. $\Delta_0 = \omega_c - \omega_L$ represents the detuning between the cavity and the driving field. The parameter g is the single-photon optomechanical coupling coefficient of each optomechanical system (assumed to be symmetrical), and J signifies the strength of photon tunneling.

The system dynamics is described by a set of quantum Langevin equations (QLEs) [36]:

$$\dot{A}_j = -(\kappa/2 + i\Delta_0)A_j - iJA_{3-j} + igA_j(B_j + B_j^\dagger) + E(t) + \sqrt{\kappa}a_j^{in}(t), \tag{2a}$$

$$\dot{B}_j = -(\gamma_{mj}/2 + i\omega_m)B_j + igA_j^\dagger A_j + \sqrt{\gamma_{mj}}b_j^{in}(t), \tag{2b}$$

where κ is the leakage rate of the cavities, and $a_j^{in}(t)$ and $b_j^{in}(t)$ are independent input vacuum noise operators obeying the following nonzero autocorrelation functions:

$$\langle a_j^{in}(t)a_j^{in\dagger}(t')\rangle = \delta(t - t'), \tag{3a}$$

$$\langle b_j^{in}(t)b_j^{in\dagger}(t')\rangle = (\bar{n}_b + 1)\delta(t - t'), \tag{3b}$$

$$\langle b_j^{in\dagger}(t)b_j^{in}(t')\rangle = \bar{n}_b\delta(t - t'), \tag{3c}$$

with $\bar{n}_b$ being the mean thermal occupancy of the mechanical bath.

In the presence of strong external driving pulses, the system operators can be written as $A_j = \alpha_j(t) + a_j$ and $B_j = \beta_j(t) + b_j$, where a_j and b_j are quantum fluctuation operators with zero mean values around classical c-number mean amplitudes $\alpha_j(t)$ and $\beta_j(t)$, respectively. Under the strong coherent driving regime $|\alpha_j(t)|, |\beta_j(t)| \gg 1$, by applying standard linearization techniques to Eq. (2), we gain a set of differential equations for the mean values:

$$\dot{\alpha}_j(t) = -(\kappa/2 + i\Delta_0)\alpha_j(t) - iJ\alpha_{3-j}(t) + ig\alpha_j(t)[\beta_j(t) + \beta_j(t)^*] + E(t), \tag{4a}$$

$$\dot{\beta}_j(t) = -(\gamma_{mj}/2 + i\omega_m)\beta_j(t) + ig|\alpha_j(t)|^2, \tag{4b}$$

and the linearized QLEs for the quantum fluctuations:

$$\dot{a}_j = -(\kappa/2 + i\Delta_0)a_j - iJa_{3-j} + ig\{a_j[\beta_j(t) + \beta_j(t)^*] + \alpha_j(t)(b_j + b_j^\dagger)\} + \sqrt{\kappa}a_j^{in}(t), \tag{5a}$$

$$\dot{b}_j = -(\gamma_{mj}/2 + i\omega_m)b_j + ig[a_j^\dagger\alpha_j(t) + a_j\alpha_j(t)^*] + \sqrt{\gamma_{mj}}b_j^{in}(t), \tag{5b}$$

which correspond to a linearized system Hamiltonian:

$$H^{\text{lin}} = \sum_{j=1,2}\{\Delta_j(t)a_j^\dagger a_j + \omega_m b_j^\dagger b_j + [G_j(t)^* a_j + G_j(t)a_j^\dagger](b_j^\dagger + b_j)\} + J(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{6}$$

with $\Delta_j(t) = \Delta_0 - g[\beta_j(t) + \beta_j(t)^*]$ and $G_j(t) = -g\alpha_j(t)$ being the effective detuning and enhanced optomechanical coupling strength induced by the driving laser, respectively.

It is difficult to find exact solutions of the mean values in Eq. (4) in general. But when we focus on the weak optomechanical coupling regime (namely, $|g/\omega_m| \ll 1$) as well as not much difference of the mean values between the two subsystems [i.e., $\alpha_1(t) \simeq \alpha_2(t) = \alpha(t)$, $\beta_1(t) \simeq \beta_2(t) = \beta(t)$], approximately analytical solutions for Eq. (4) can be found by expanding the classical mean values $\alpha(t)$ and $\beta(t)$ in powers of g as [30,31,37]

$$\alpha(t) = \alpha(t)^{(0)} + \alpha(t)^{(1)} + \alpha(t)^{(2)} + \cdots, \tag{7a}$$

$$\beta(t) = \beta(t)^{(0)} + \beta(t)^{(1)} + \beta(t)^{(2)} + \cdots. \tag{7b}$$

When two four-tone driving lasers $E(t) = \sum_{l=1}^4 E_l e^{-i\omega_l t}$ are implemented, following the same procedures as Ref. [34], the asymptotic solutions for time $t \gg 1/\kappa, 1/\gamma_{m1}, 1/\gamma_{m2}$ can be obtained:

$$\alpha(t)^{(0)} = \sum_{l=1}^4 \overline{\alpha}_l e^{-i\omega_l t}, \ \beta(t)^{(0)} = 0, \tag{8}$$

where

$$\overline{\alpha}_l = E_l/[\kappa/2 + i(\Delta_0 + J - \omega_l)]. \tag{9}$$

One can further make the approximations $\alpha(t) \simeq \alpha(t)^{(0)}$ and $\Delta(t) \simeq \Delta_0$ in view of $\alpha(t)^{(1)} = 0$, $|\alpha(t)^{(2)}| \ll |\alpha(t)^{(0)}|$, and $|g[\beta(t) + \beta^*(t)]| \ll \Delta_0 \sim \omega_m$. Then the Hamiltonian of Eq. (6) in the asymptotic regime becomes

$$H_{\rm asy}^{\rm lin} = \sum_{j=1,2} \{\Delta_0 a_j^\dagger a_j + \omega_m b_j^\dagger b_j + [\overline{G}(t)^* a_j + \overline{G}(t) a_j^\dagger](b_j^\dagger + b_j)\} + J(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{10}$$

where

$$\overline{G}(t) = -g\alpha(t)^{(0)} = \sum_{l=1}^{4} \overline{G}_l e^{-i\omega_l t} \tag{11}$$

with

$$\overline{G}_l = -g\overline{\alpha}_l. \tag{12}$$

Introducing the new bosonic modes,

$$\begin{aligned} c_1 &= (a_1 + a_2)/\sqrt{2}, c_2 = (a_1 - a_2)/\sqrt{2},\\ d_1 &= (b_1 + b_2)/\sqrt{2}, d_2 = (b_1 - b_2)/\sqrt{2}, \end{aligned} \tag{13}$$

Eq. (10) becomes

$$H_{\rm asy}^{\rm lin} = \sum_{j=1,2} \{\Delta_j c_j^\dagger c_j + \omega_m d_j^\dagger d_j + [\overline{G}(t)^* c_j + \overline{G}(t) c_j^\dagger] \times (d_j^\dagger + d_j)\}, \tag{14}$$

where $\Delta_1 = \Delta_0 + J$, $\Delta_2 = \Delta_0 - J$. In the interaction picture of $\Delta_j c_j^\dagger c_j + \omega_m d_j^\dagger d_j$, Eq. (14) can be rewritten as

$$H^{\rm int} = \sum_{j=1,2} [\overline{G}(t)^* c_j e^{-i\Delta_j t} + \overline{G}(t) c_j^\dagger e^{i\Delta_j t}] \times (d_j^\dagger e^{i\omega_m t} + d_j e^{-i\omega_m t}). \tag{15}$$

To obtain the targeted Hamiltonian, we select the modulating frequencies ω_l as

$$\omega_1 = \Delta_0 + J - \omega_m, \tag{16a}$$
$$\omega_2 = \Delta_0 + J + \omega_m, \tag{16b}$$
$$\omega_3 = \Delta_0 - J - \omega_m, \tag{16c}$$
$$\omega_4 = \Delta_0 - J + \omega_m, \tag{16d}$$

and then make the rotating-wave approximation by neglecting all fast oscillating terms under the conditions $J > 2\omega_m$ and $\omega_m \gg \overline{G}_l$ to gain the effective Hamiltonian,

$$\begin{aligned} H_{\rm eff} &\simeq [\overline{G}_1 d_1 + \overline{G}_2 d_1^\dagger] c_1^\dagger + [\overline{G}_3 d_2 + \overline{G}_4 d_2^\dagger] c_2^\dagger + \text{H.c.}\\ &= \frac{1}{2}[(\overline{G}_1 + \overline{G}_3) b_1 + (\overline{G}_1 - \overline{G}_3) b_2 + (\overline{G}_2 + \overline{G}_4) b_1^\dagger\\ &\quad + (\overline{G}_2 - \overline{G}_4) b_2^\dagger] a_1^\dagger + \frac{1}{2}[(\overline{G}_1 - \overline{G}_3) b_1 + (\overline{G}_1 + \overline{G}_3) b_2\\ &\quad + (\overline{G}_2 - \overline{G}_4) b_1^\dagger + (\overline{G}_2 + \overline{G}_4) b_2^\dagger] a_2^\dagger + \text{H.c.} \end{aligned} \tag{17}$$

When $\overline{G}_l$ satisfies

$$\overline{G}_2 = -\overline{G}_4 = G_+, \overline{G}_1 = \overline{G}_3 = G_-, |G_-| > |G_+|, \tag{18}$$

we finally obtain the following Hamiltonian,

$$H_{\rm eff} = \widetilde{G}(\theta_1 a_1^\dagger + \theta_2 a_2^\dagger) + \text{H.c.}, \tag{19}$$

where $\widetilde{G} = \sqrt{G_-^2 - G_+^2}$ and the introduced Bogoliubov modes θ_1 and θ_2 are unitary transformations of the mechanical modes b_1 and b_2 with a two-mode squeezed operator, respectively,

$$\theta_1 = S(r) b_1 S^\dagger(r) = b_1 \cosh r + b_2^\dagger \sinh r, \tag{20a}$$
$$\theta_2 = S(r) b_2 S^\dagger(r) = b_2 \cosh r + b_1^\dagger \sinh r, \tag{20b}$$
$$S(r) = \exp[r(b_1 b_2 - b_1^\dagger b_2^\dagger)], \tag{20c}$$
$$r = \tanh^{-1}(G_+/G_-). \tag{20d}$$

Note that Eq. (19) is a beam-splitter-like Hamiltonian, which is well known from optomechanical sideband cooling [38,39]. For the small mechanical damping rate considered here, the dissipation of the cavity modes a_1 and a_2 can be exploited to simultaneously cool the Bogoliubov modes θ_1 and θ_2 to near ground states after long enough time. The joint ground state of θ_1 and θ_2 is a two-mode squeezed vacuum state of the mechanical modes b_1 and b_2, which can be readily checked by

$$\theta_j[S(r)|00\rangle_{b_1 b_2}] = S(r) b_j S^\dagger(r) S(r)|00\rangle_{b_1 b_2} = 0. \tag{21}$$

In the above analysis, two cavity modes are used as two engineered reservoirs to cool the two mechanical modes to a two-mode squeezed state in the stationary limit. The two-mode squeezed state is an entangled state and has quantum steering. By selecting different damping rates of the two mechanical modes, the whole system will be asymmetric, and it is possible to observe one-way quantum steering. In the next section, we will confirm the existence of the one-way quantum steering in the system by numerical simulation.

III. QUANTUM STEERING AND ENTANGLEMENT

In the following, we investigate in detail the properties of stationary quantum steering and analyze how to obtain one-way quantum steering by controlling the amplitudes of external lasers and choosing an appropriate ratio of two mechanical damping rates. By introducing the position and momentum quadratures for the bosonic operators ($o \in \{a_j, b_j, a_j^{\rm in}, b_j^{\rm in}\}$),

$$q_o = (o + o^\dagger)/\sqrt{2},\ \ p_o = (o - o^\dagger)/(i\sqrt{2}), \tag{22}$$

and the column vectors of all quadratures and noises,

$$R = (q_{b_1}, p_{b_1}, q_{b_2}, p_{b_2}, q_{a_1}, p_{a_1}, q_{a_2}, p_{a_2})^T, \tag{23a}$$

$$N = \big(\sqrt{\gamma_{m1}}q_{b_1^{in}}, \sqrt{\gamma_{m1}}p_{b_1^{in}}, \sqrt{\gamma_{m2}}q_{b_2^{in}}, \sqrt{\gamma_{m2}}p_{b_2^{in}}, \sqrt{\kappa}q_{a_1^{in}}, \sqrt{\kappa}p_{a_1^{in}}, \sqrt{\kappa}q_{a_2^{in}}, \sqrt{\kappa}p_{a_2^{in}}\big)^T, \tag{23b}$$

Eq. (5) can be transformed into a more compact form,

$$\dot{R} = MR + N, \tag{24}$$

with

$$M = \begin{pmatrix} -\gamma_{\rm m1}/2 & \omega_{\rm m} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\omega_{\rm m} & -\gamma_{\rm m1}/2 & 0 & 0 & -2G_{1\rm R}(t) & -2G_{1\rm I}(t) & 0 & 0 \\ 0 & 0 & -\gamma_{\rm m2}/2 & \omega_{\rm m} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_{\rm m} & -\gamma_{\rm m2}/2 & 0 & 0 & -2G_{2\rm R}(t) & -2G_{2\rm I}(t) \\ 2G_{1\rm I}(t) & 0 & 0 & 0 & -\kappa/2 & \Delta_1(t) & 0 & J \\ -2G_{1\rm R}(t) & 0 & 0 & 0 & -\Delta_1(t) & -\kappa/2 & -J & 0 \\ 0 & 0 & 2G_{2\rm I}(t) & 0 & 0 & J & -\kappa/2 & \Delta_2(t) \\ 0 & 0 & -2G_{2\rm R}(t) & 0 & -J & 0 & -\Delta_2(t) & -\kappa/2 \end{pmatrix}, \tag{25}$$

where $G_{jR}(t)$ and $G_{jI}(t)$ are respectively real and imaginary parts of the effective coupling constant $G_j(t)$. Since the dynamics of the four-mode bosonic system is governed by a linearized Hamiltonian, the system converges to a time-dependent Gaussian state when it is stable [40], which is independent from its initial state. The asymptotic state is fully described by the covariance matrix (CM) σ with entries defined as

$$\sigma_{j,l} = \langle R_j R_l + R_l R_j \rangle/2, \tag{26}$$

where R_j is the jth component of the vector R of quadratures.

From Eqs. (3), (23), and (24), a linear differential equation

$$\dot{\sigma} = M\sigma + \sigma M^T + D \tag{27}$$

for the CM can be deduced [41], where D is a diffusion matrix whose components are associated with the input noise correlation functions in Eq. (3),

$$D_{j,l}\delta(t - t') = \langle N_j(t)N_l(t') + N_l(t')N_j(t)\rangle/2. \tag{28}$$

Actually, one can find that D is diagonal,

$$D = \frac{1}{2} \times {\rm diag}[\gamma_{\rm m1}(2\bar{n}_{\rm b} + 1), \gamma_{\rm m1}(2\bar{n}_{\rm b} + 1), \gamma_{\rm m2}(2\bar{n}_{\rm b} + 1), \gamma_{m2}(2\bar{n}_b + 1), \kappa, \kappa, \kappa, \kappa]. \tag{29}$$

In the following, we will utilize Eq. (27) to study the time evolution of the quantum steering and entanglement of the two mechanical modes. Note that the coefficient matrix in Eq. (25) corresponds to the system Hamiltonian in Eq. (6), where the only approximation is the commonly used linearization techniques in optomechanics.

For two-mode Gaussian states of the two mechanical resonators b_1 and b_2, a computable criterion of quantum steering based on the form of quantum coherent information has been introduced [10]. As for quantum entanglement, it is convenient to use the logarithmic negativity $E_{\rm N}$ to gauge its level [42,43]. All the above-mentioned measures can be computed from the reduced 4×4 CM $\sigma_{\rm r}(t)$ for b_1 and b_2 (i.e., the first four rows and columns of CM),

$$\sigma_{\rm r}(t) = \begin{pmatrix} \sigma_1 & \sigma_{\rm c} \\ \sigma_{\rm c}^{\rm T} & \sigma_2 \end{pmatrix}, \tag{30}$$

where σ_1, σ_2, and $\sigma_{\rm c}$ are 2×2 sub-block matrices of $\sigma_{\rm r}(t)$. The entanglement is then calculated by

$$E_{\rm N} = \max[0, -\ln(2\eta)] \tag{31}$$

with

$$\eta \equiv 2^{-1/2}\{\Sigma_- - [\Sigma_-^2 - 4I_4]^{1/2}\}^{1/2} \tag{32}$$

and

$$\Sigma_- \equiv I_1 + I_2 - 2I_3, \tag{33}$$

where $I_1 = \det\sigma_1$, $I_2 = \det\sigma_2$, $I_3 = \det\sigma_{\rm c}$, and $I_4 = \det\sigma_{\rm r}$ are the symplectic invariants. The Gaussian $b_1 \to b_2$ steering is given by

$$G_A \equiv G^{b_1\to b_2}(\sigma_{\rm r}) = \max\left[0, \frac{1}{2}\ln\frac{I_1}{4I_4}\right], \tag{34}$$

and a corresponding measure of Gaussian $b_2 \to b_1$ steerability can be obtained by swapping b_1 and b_2, resulting in an expression

$$G_B \equiv G^{b_2\to b_1}(\sigma_{\rm r}) = \max\left[0, \frac{1}{2}\ln\frac{I_2}{4I_4}\right]. \tag{35}$$

In order to check the asymmetric steerability of two-mode Gaussian states, we introduce the steering asymmetry defined as $|G_A - G_B|$.

The numerical results are shown in Figs. 2 and 3. All results are numerically evaluated with the full linearized QLEs for the quantum fluctuations Eq. (5) [or equally, the linear differential Eq. (27) for the CM], including the nonresonant terms and with all mechanical and cavity modes initially in thermal equilibrium with their baths. To do this, we first numerically integrate the differential Eq. (4) by applying two four-tone driving lasers with weighted amplitudes and specific frequencies $E(t) = \sum_{l=1}^{4} E_l e^{-iw_l t}$ to get the time-dependent

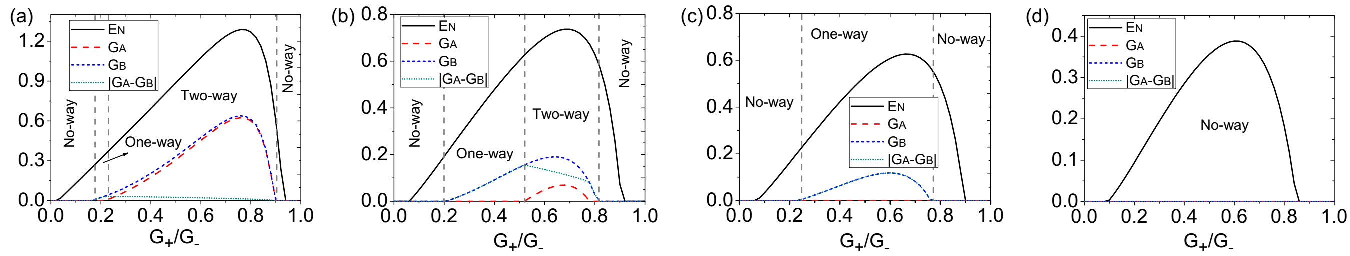

FIG. 2. Maximum mechanical-mechanical entanglement and steering for each time period in the asymptotic regime as functions of the ratio of the effective coupling G_+/G_- for different damping rate ratios γ_{m2}/γ_{m1}: (a) $\gamma_{m2}/\gamma_{m1} = 2$, (b) $\gamma_{m2}/\gamma_{m1} = 8$, (c) $\gamma_{m2}/\gamma_{m1} = 10$, and (d) $\gamma_{m2}/\gamma_{m1} = 16$. The other parameters are $\kappa/\omega_m = 0.1$, $\gamma_{m1}/\omega_m = 1 \times 10^{-4}$, $\Delta/\omega_m = 4$, $J/\omega_m = 3$, $g/\omega_m = 1 \times 10^{-5}$, $G_-/\omega_m = 0.03$, and $\bar{n}_b = 5$.

mean values for Eqs. (5) and (6), using a set of experimentally achievable parameters [44–47].

Figure 2 displays the peak values of stationary entanglement and steering of the two mechanical modes for each time period in the long-time limit as functions of the coupling asymmetry G_+/G_-, where different mechanical damping rate ratios γ_{m2}/γ_{m1} are used in different subplots. The value G_+/G_- determines the squeezed vacuum state that can be cooled down by using two cavity modes as two engineered reservoirs [see Eqs. (19) and (20)]. As has been previously studied in Refs. [31–35], the increase of the coupling asymmetry G_+/G_- has two competing effects. On the one hand, it can increase the squeezing parameter $r = \tanh^{-1}(G_+/G_-)$ of the two-mode squeezed thermal state. On the other hand, it will reduce the effective coupling $\widetilde{G} = \sqrt{G_-^2 - G_+^2}$ between the Bogoliubov modes θ_j and the cavity mode a_j, which is harmful for the cooling effect. Therefore, entanglement and steering are nonmonotonic functions of G_+/G_- and take a maximum for a specific G_+/G_-. From the subplots of Fig. 2, it can be found that the optimal values of both steering and entanglement (especially for the $b_1 \to b_2$ steering G_A) gradually decrease with the increase of the damping rate γ_{m2}. Peculiarly, when the damping rate γ_{m2} is large enough [e.g., $\gamma_{m2}/\gamma_{m1} = 10$ in Fig. 2(c)], the $b_1 \to b_2$ steering G_A disappears completely, while there is still $b_2 \to b_1$ steering G_B. As the damping rate γ_{m2} gets larger [e.g., $\gamma_{m2}/\gamma_{m1} = 16$ in Fig. 2(d)], the $b_2 \to b_1$ steering G_B also vanishes while the optimal entanglement obtained is still large enough, which proves again that the steering is a nonclassical correlation stronger than entanglement. The above observed phenomena can be explained as follows. When the mechanical decay rate γ_{m2} is larger, there is a stronger interaction between the mechanical mode b_2 and its thermal bath, which raises the final effective temperature of Bogoliubov modes (especially the mechanical mode b_2). Since both quantum steering and entanglement are sensitive to the environmental temperature, their optimal values gradually decrease with the increase of the damping rate γ_{m2}. Specifically, due to the asymmetry of the system, the $b_1 \to b_2$ steering G_A drops faster than the $b_2 \to b_1$ steering G_B. At first sight, this result seems to be inconsistent with earlier studies [14,24,48,49], but it actually is not. The result comes from the fact that the raised effective temperature of the mechanical mode b_2 will enlarge its corresponding quantum fluctuations. That is to say, the steerability between two mechanical modes is simultaneously affected, both by the mode damping (loss) and thermal bath (noises), and the fact that the mechanical mode with larger damping rate is more difficult to steer by the other one when the mean thermal occupancy of the mechanical baths are not negligible. Therefore, when certain conditions are met, there exist entangled states which are $b_2 \to b_1$ one-way steerable. The states that have $b_2 \to b_1$ one-way steering are clearly shown in Figs. 2(b) and 2(c). The $b_2 \to b_1$ one-way steering implies that Bob can convince Alice that their shared state is entangled, while the converse is not true. The most obvious application is that it provides security in one-sided device-independent quantum key distribution (QKD), where the measurement apparatus of one party only is untrusted.

Figure 3 displays the peak values of stationary entanglement and steering of the two mechanical modes for each time

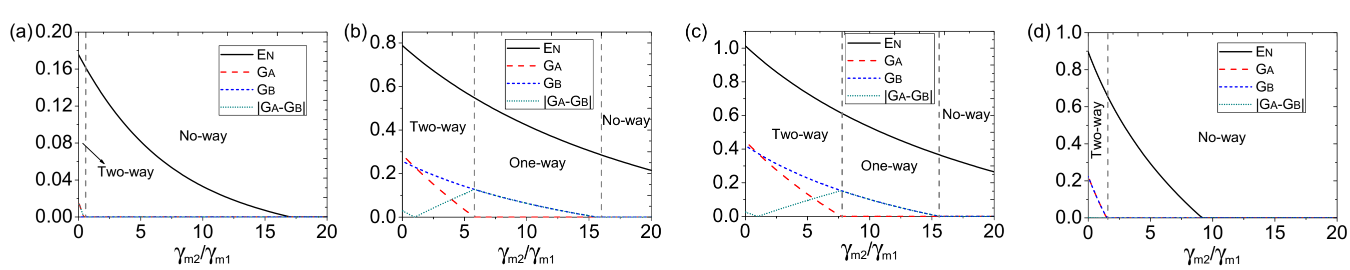

FIG. 3. Maximum mechanical-mechanical entanglement and steering for each time period in the asymptotic regime as functions of the ratio γ_{m2}/γ_{m1} of the damping rate for different effective coupling ratios G_+/G_-: (a) $G_+/G_- = 0.1$, (b) $G_+/G_- = 0.4$, (c) $G_+/G_- = 0.5$, and (d) $G_+/G_- = 0.9$. The other parameters are $\kappa/\omega_m = 0.1$, $\gamma_{m1}/\omega_m = 1 \times 10^{-4}$, $\Delta/\omega_m = 4$, $J/\omega_m = 3$, $g/\omega_m = 1 \times 10^{-5}$, $G_-/\omega_m = 0.03$, and $\bar{n}_b = 5$.

period in the long-time limit as functions of the ratio γ_{m2}/γ_{m1}, where different ratios G_+/G_- are used in different subplots. The value γ_{m2}/γ_{m1} determines the asymmetry of the system. From Fig. 3 it can be found that the entanglement and the steering all drop rapidly with the increase of γ_{m2}/γ_{m1}, and the $b_1 \rightarrow b_2$ steering G_A usually vanishes earlier than the $b_2 \rightarrow b_1$ steering G_B. Therefore there are states that only have one-way quantum steering.

For the sake of gaining a large one-way steering while retaining the relatively high amount of entanglement, we must select proper ratio of effective coupling G_+/G_- and the damping rate ratio γ_{m2}/γ_{m1}. From Eqs. (9), (12), and (18), G_+/G_- is proportional to the ratio of the driving amplitudes E_2/E_1 when other system parameters remain fixed. Thus, one-way steering can be observed by controlling the amplitudes of external lasers and choosing an appropriate ratio of two mechanical damping rates.

Finally, it should be noted that the goal can in principle be achieved by using two two-tone driving lasers with a two-step process [30]. However, in our scheme we simultaneously make two modes in the squeezed state with two four-tone lasers by only one step. As a consequence, the effect of the mechanical damping in our scheme may be smaller. The four-tone driving lasers have been used in the theoretical work of Ref. [32] and the related experimental work of Ref. [29]. In Ref. [29], two driving tones are used to prepare the two-mode squeezed state, while two other driving tones are used to measure the state prepared. Hence, the four-tone lasers can be implemented in principle under the current experimental conditions.

IV. CONCLUSIONS

In summary, we have investigated an optomechanical system consisting of two cavity modes and two mechanical modes. The two cavity modes are pumped by two four-tone driving lasers. The numerical simulation results reveal that both steering and entanglement of the two mechanical modes can be greatly enhanced by controlling the amplitudes and frequencies of the pumping lasers. Specifically, when we increase the damping rate γ_{m2}, it is found that the $b_1 \rightarrow b_2$ steering vanishes earlier than the $b_2 \rightarrow b_1$ steering, which means that the two mechanical modes can be in a state that has only one-way quantum steering. One-way quantum steering may have potential applications in the quantum information protocols, such as device-independent quantum key distribution. Our one-way quantum steering generation method is a reservoir-engineered method that does not depend on the initial states of the system, which may reduce the experimental realization requirement.

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